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ORIGINAL PAPER

Geometric-distortions and physical structure modeling

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Abstract: An investigation on the interpretation of localized geometric distortions as a source of microscopic physical particle-structures has been undertaken. Modeling utilizes the fundamental curvature equations of Riemannian geometry. It is limited to geometric-distortional families satisfying a simple temporal-to-spatial tensor equation-of-state, a Maxwellian relationship. A microscopic-level coupling-constant also results, which supplants and extends the classical gravitational coupling-constant. The geometrical tensor elements of the distorted space exhibit negative, as well as positive, curvature-magnitudes and energy-densities while the field-observable in the negative energy–density spatial core-region is non-Coulombic and non-infinite at the radial origin. Fundamental particles and gravitational structures have been mimicked.

Keywords: Geometry; Classical field theory; General relativity; Classical mechanics; Structural stability

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1. Introduction

In the present undertaking, we maintain geometrical perspectives inherent in "Curved empty space as the building material of the physical world" supposition [1–4]. We posit therefore that the classical Riemannian four-dimensional geometrical curvature equations can be applied to describe distortional-energy-distributions at elementary-particle-level magnitudes and distances by utilizing a geometric equation-of-state and a modified-gravitational model-derived coupling-constant between geometry and physical energy. The treatment supposes the fundamental notion that local spatial distensions within the four-dimensional space–time manifold, which are describable by the geometrical curvature relationships, are localized stable and metastable energy configurations which can mimic the physical characteristics of the elementary particles. The present phenomenological modeling fundamentally constrains the character of the associated coupling constant and determines the required values to fit the geometric-distortion descriptors to the physical characteristics of the modeled particles.

The functional solutions to the differential equations describing the "distorted" space are of such a character that, over portions of the radial extension of the distortion, the geometrical tensor elements exhibit negative (in the core-regions), as well as positive, curvature-magnitudes [5]. These are "energy–density" quantities (converted via the coupling constant κ (m/J)) and both the "mass energy–density" ($Td_4^4 - Td_1^1 - 2Td_2^2$) and the "field-tensor energy–density sums", $Td_4^4 + Td_1^1$ for an "electric-field" and $Td_4^4 - Td_1^1$ for a "magnetic-field", therefore exhibit spatial regions of negative "energy–density". For the "mass–energy", however, a spatial integral, the result for the total summation ($r = 0 \rightarrow \infty$) is a "positive" quantity, a "positive mass-energy". Although the mass energy–density spatial distribution may not be physically measurable, the "field-observable" in the "negative energy–density" spatial region (the core region) is non-Coulombic and is manifested at magnitudes significantly different than the core-region Coulomb values.

We also consider a distortional, or particle, transformation process wherein the process is described in terms of a quantized geometrical mediating particle with characteristics mimicking the Fermi β-decay transition.

A comprehensive and general treatment of the historical, geometric and physical foundations of modern geometrodynamics is already reported [1–3]. Additional work in this field continues, some of which is cited in references [6–11]. The present treatment departs from these cited "geon constructional methods" in that we do not constrain the distortional descriptions to only gravitational coupling-constant produced structures.

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2. Structural equations and physical modeling

The geometrical model begins with the classical curvature representation of a spatial energy distribution defining the particle. Einstein's gravitational equations [12] are qualified on a microscopic scale to an isotropic space and are modified to unspecify the geometrically-defined coupling-constant, κ, between geometry and physical energy.

$$G_{ab}(g_{ab}) \equiv R_{ab} - \frac{1}{2} g_{ab} R = 8\pi\kappa T_{ab}. \tag{1}$$

G_{ab} is the Einstein Curvature tensor, a function of the metric g_{ab} and its first two derivatives, R_{ab} is the Ricci tensor and R, the Ricci scalar. T_{ab}, which classically is the stress-energy-tensor describing the material contents of the particle energy distribution, is here viewed as the distortionally-generated geometric-tensor, with its associated energy content resulting from the creation process which produced the distorted feature in the manifold; the energy is fundamentally manifested in the distorted metric itself. The "curved empty space" [4] referred to above is a "curved space" devoid of an "external or foreign" causative matter-entity. The functional form of the spherically-symmetric distorted-metric elements g_{11} and g_{44} appearing in the line element describing the distorted geometric-region, or Eq. (2), is to be determined.

$$\begin{aligned} ds^2 &= g_{11}[dr^2 + r^2 d\Omega^2] + g_{44} dt^2 \\ &= -e^{-\mu}[dr^2 + r^2 d\Omega^2] + e^{\nu} dt^2, \end{aligned} \tag{2}$$

with μ = μ(r, t) and $\boldsymbol{\nu} = \boldsymbol{\nu}$(r, t).

Tolman [13] has shown that the energy of a "quasi-static isolated system" can be expressed as "an integral extending only over the occupied space", which we have allowed to extend to infinity, and where the total energy of such a sphere is therefore expressed as

$$\begin{aligned} U_{sphere\ total} = \int_0^{\infty} & \left(T_4^4 - T_1^1 - T_2^2 - T_3^3\right) \\ & \times \sqrt{\left(-g_{11}^3\right) g_{44}}\, 4\pi r^2 dr = M_{sphere} c^2. \end{aligned} \tag{3}$$

This mass-energy representation has been used throughout in calculating the distortional mass-energies. We constrain the modeling by requiring that the descriptive stress-energy tensors satisfy a "constitutive relation" or an "equation-of-state" between the temporal and spatial tensor-curvature elements, namely $Td_4^4 = -(Td_1^1 + Td_2^2 + Td_3^3)$. We have introduced the explicit distortional-tensor symbolism Td for the geometric quantities.

In using this simple equation-of-state as a restricting distortional-model tensor relationship, we thereby elicit the metric-defining differential equations for such a family of geometric distortions and in the current procedural characterization, one has sufficient information to move to a solution of the differential equations without explicitly stating any "material" energy densities, thereby maintaining the spatial-distortion causative-perspective. In generalized format we construct a temporal-to-spatial metric relationship as $\nu' = [-2 + f(r)]\mu'$ (conveniently simple) with $f(r)$ further defined as,

$$f(r) \equiv (1 - u(r)^n)^{-1} \quad \text{with } u(r) \equiv (R_0/r). \tag{4}$$

R_0 is a distortional-characteristic, or normalizing, radius. In the following development we utilize n = 3 for the modeled particles. The system of equations represented by Eq. (1), while maintaining the use of relationship given by Eq. (4), is shown in mixed tensor form for a static system as

$$\begin{aligned} 8\pi\kappa Td_1^1 &= -e^{-\mu}\mu'\left[\frac{\mu'}{4}(2f-3) - \frac{1-f}{r}\right], \\ 8\pi\kappa Td_2^2 &= -\frac{e^{-\mu}}{2}\mu'\left[\frac{1}{f}f' + \frac{1}{2f}(3-2f)\mu' + \frac{1-f}{r}\right] = 8\pi\kappa Td_3^3, \\ 8\pi\kappa Td_4^4 &= e^{-\mu}\mu'\left[\frac{1}{f}f' + \frac{\mu'}{4f}(2f-3)(f-2)\right]. \end{aligned} \tag{5}$$

After solving for μ and subsequently forming the asymptotic metric form, that is, the expanded large-radius metric, we have equated the distortional-geometric form to a Schwarzschild form with the result that $2/C1 = -Rs$.

Imposing the equation-of-state relationship, the metric differential equation is

$$\mu'' + \frac{f'}{f}\mu' + \frac{1}{2f}[(f-3)(f-1) + f](\mu')^2 + \frac{2\mu'}{r} = 0. \tag{6}$$

A solution to this equation is found to be

$$\begin{aligned} &\mu' = \tfrac{2}{r^2 f} g(r) \quad \text{with } g(r)^{-1} = \int \tfrac{(f-3)(f-1)+f}{r^2 f^2} dr + C1 \equiv \tfrac{Iu}{R0} + C1, \\ &\text{or with } u \equiv u(r), \quad \mu' = \tfrac{2(1-u^3)}{Iu-\gamma}\tfrac{u^2}{R0} \quad \text{where } \gamma \equiv -C1R0 \\ &\text{and where } Iu(u) = u\left[-1 + \tfrac{3}{4}u^3 - \tfrac{3}{7}u^6\right]. \end{aligned} \tag{7}$$

We examine the geometrostatic field quantities mimicking the Maxwellian field quantities $(F_{14})^2$ and $(F_{12})^2$. We do so by comparing $(F_{14})^2$ and $(F_{12})^2$ at "classical large-radii regions" with the geometrostatic field quantities of Eq. (8).

$$\begin{aligned} &Fd_{14}^2 = -g_{11}g_{44}\left[Td_4^4 + Td_1^1\right] \text{ and} \\ &Fd_{14}^2(r \to \infty) \cong \frac{1}{C1^2}\frac{2}{8\pi\kappa}\frac{1}{r^4} \equiv \left(\frac{q}{4\pi\varepsilon o r^2}\right)^2 \frac{\varepsilon o}{2}. \\ &Fd_{12}^2 + Fd_{13}^2 = 2g_{11}g_{11}\left[\frac{Td_4^4 - Td_1^1}{2}\right] \equiv Fd_{mag}^2 \text{ and} \\ &Fd_{mag}^2(r \to \infty) \cong -4\frac{R0^3}{C1}\frac{1}{8\pi\kappa}\frac{1}{r^6} \equiv \frac{\mu o}{2}\left(\frac{\mu\_spin}{2\pi}\right)^2 \frac{1}{r^6} \\ &where\ \mu_{spin} \equiv \left(\frac{g_e}{2}\frac{Qe}{3M}\right) \mathrm{S}\frac{h}{2\pi} \text{ and } g_e = 2.00231930436. \end{aligned} \tag{8}$$

For this geometrostatic case, the $(Fd_{12})^2$ and $(Fd_{13})^2$ field quantities are identical and therefore considered functionally

addable, at the tensor expression level, to form the single field quantity $(Fd_{mag})^2$. Although there are no "moving" charges in this static spherically symmetric model, the geometric field tensor entity $(Fd_{mag})^2$ is non-zero and therefore both "pseudo-electrostatic" and "pseudo-magnetostatic" distortional-fields are produced. Spherical symmetry precludes azimuthal and polar fields and therefore we are mimicking an r-dependent, r-directed field. The "pseudo-magnetostatic" distortional-field source-strength is that of a "magnetic-dipole of magnitude to produce the dipole-axial-field". The electrostatic and magnetostatic constraints, which we constructed at large radii with r^{-4} and r^{-6} behaviors, have therefore required for mimicking, that the coupling constant be a variable; $h =$ Planck's constant, $q =$ particle electric-charge, $M =$ particle mass, $\mu o =$ permeability constant, $c =$ speed of light $\equiv$ the flat-space propagation-velocity within the geometric manifold. Having mimicked both mass-energy and electromagnetic-energy behavior, the coupling constant is a function of both source entities, the electric charge and the mass, but in a combined fashion expressed through the single physical descriptor μ_spin;

$$\kappa o = \frac{\alpha fs \frac{hc}{2\pi}\left(\frac{Q}{3}\right)^2}{2(Mc^2)^2} = \frac{\mu o}{2\pi}\left(\frac{\mu\_spin}{\frac{hc}{2\pi}Sg_e}\right)^2,$$

$C1 = -2/Rs = -1/(\kappa o Mc^2)$, and

$$R0^3 = \left(\frac{hc}{2\pi}Sg_e\right)^2 \frac{\kappa o}{2Mc^2} = \frac{\mu o}{4\pi}\frac{\mu\_spin^2}{Mc^2} = \frac{Lf^4}{Rs}(Sg_e)^2, \qquad (9)$$

where Q (quantized) $= \pm1, \pm2,$ or ±3, $e \equiv$ electron charge, $\alpha fs \equiv$ fine structure constant, $g \equiv$ particle g-factor and $S \equiv$ particle spin-factor. Although a classical presentation would not require a "quantization" restriction, we have included this feature to reflect experimental reality. The quantity $Lf \equiv \sqrt{\hbar c \kappa o}$, in conjunction with a gravitational coupling constant, (i.e. $\kappa o \equiv \kappa_G \equiv G/c^4$) has heretofore been posited as a vacuum-fluctuation radius [2].

In summary, Eqs. (2)–(9) form the equation set for describing the distortional family's "particle-like" characteristics and its associated Maxwellian-like fields.

We calculate the elemental metric quantities, the associated metrics and the propagation velocity or null geodesic $((-g_{11})^{-1}g_{44})^{0.5}$, with the dimensionless results displayed in Fig. 1 for the modeled particle 2. Within the "core regions" of the distortion, the propagation velocity exceeds the velocity of light (equivalent to an $n < 1$ refractive index). We illustrate in Fig. 2 the field results calculated according to Eqs. (5) and (8) with the geometric distortion tensors replacing the Maxwellian tensors. The solution in the core region produces a field variable significant in the fact that it does not exhibit a singularity at the radial origin. For comparison purposes, we have also shown the classical

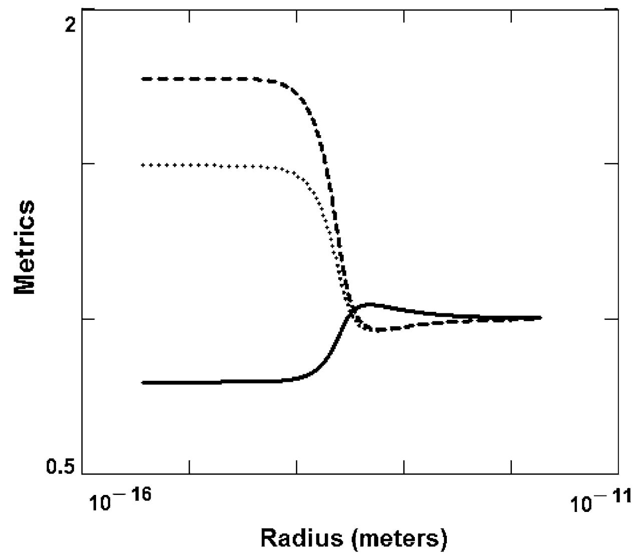

Fig. 1 Metrics g_{11} represented by *continuous line*, g_{44} represented by *discrete line* and propagation-velocity factor represented by *dotted line* for a distortion-2 (electron) structure as a function of the radius measured from the distortion center

Maxwellian electrostatic field quantity in energy–density form, $u_E = (E_{Max})^2\varepsilon o/2$. In the core region the geometric fields also differ notably from the Maxwellian fields in that they exhibit a change of sign relative to the extra-core region and are therefore strongly non-Coulombic (Fig. 2).

The classical Maxwellian field-constructs have been here geometrically generalized to permit the negative energy–density stress-tensor-based functions to still constitute the sources of the fields; we have posited and used therefore, for example, that

$$\pm\frac{1}{2}\varepsilon o E^2 \equiv \pm Fd^2 \equiv \pm \mathrm{u} \quad \text{or,}$$

$$E(u > 0) = \pm\sqrt{+u} \quad \text{and} \quad E(u<0) = \pm\sqrt{-u}, \qquad (10)$$

where $(Fd)^2$ is the geometric-field construct $(Fd_{14})^2$, $(Fd_{12})^2$ or $(Fd_{13})^2$ according to Eqs. (5) and (8). The positive "field-sign" for $E(\mathrm{u} > 0)$ is associated with a positive "electric-charge" and the negative "field-sign" for $E(\mathrm{u} < 0)$ is associated with a positive "electric-charge".

Table 1 summarizes the relevant features of the geometrostatic distortions and the simulated particles where the particle masses are taken from [14]. The results for the "sphere mass-energy", Eq. (3), are also indicated in Table 1 where $U \equiv U(sphere)$.

Finally, to further aid in understanding or grasping the spatial character of these distortions, we form a supplementary pictorial representation of their geometric structural details and display qualitative surface-plots (Figs. 3 and 4) of the mass-energy–density functions for the particle-2 distortion (two views labeled ρMass_e).

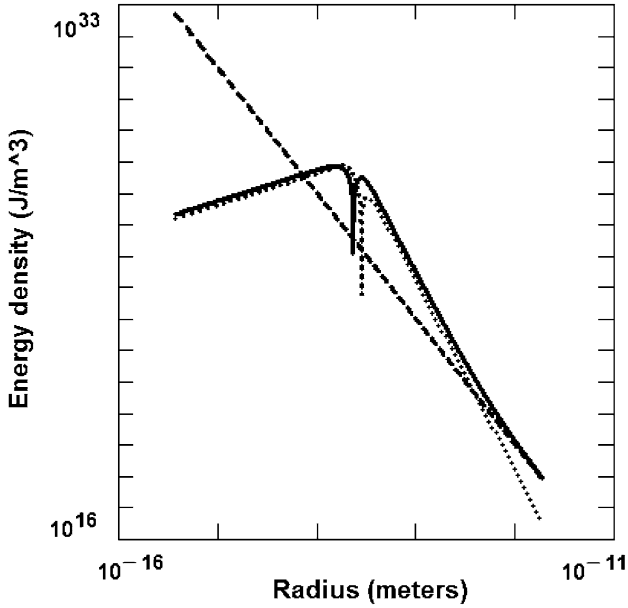

Fig. 2 Field-energy–density distribution functions (electric and magnetic) for a distortion-model 2 (electron) structure where $(Fd_{14})^2$ is represented by *continuous line*, $(Fd_{mag})^2$ is represented by *dotted line* and $u_E = (E_{Max})^2 \varepsilon o/2$ is represented by *discrete line*. The distribution functions are illustrated as a function of the radius measured from the distortion center

3. Distortional transition processes

In Fermi's characterization of the beta decay [15] particle-restructuring process, now described as an "electroweak interaction" phenomenon, we see a characterization of the strength of the process quoted as a product-function involving an "energy (field)" factor and an associated "spatial volume" factor. For the geometric domain we interpret such a characterization as a "structural description" of the distortional-feature which mediates the restructuring or decay process. The mimicking constraint on the geometrical entity therefore is that it must exhibit the physical features characteristic of the Fermi (Universal Fermi Interaction) constant, *GF*.

These constraining criteria are satisfied as represented in the earlier Eq. (9) and the following Eq. (11a) and (11b). In the development of Eq. (11a) and (11b), it is seen that the geometric-mimic of the magnetostatic energy–density form is the fundamental source of the functional description of the mimicked Fermi function. We use the designation $FM \equiv$ Fermi_Mass $= m_F me\ c^2$ for the transition-distortion mass-energy and on setting the Fermi constant equal to the geometric product (volume × mass-energy), we produce Eq. (11a) and (11b). We use

$$\frac{R0^3 Mc^2}{2\pi} = \frac{\mu o}{2}\left(\frac{\mu_{spin}}{2\pi}\right)^2 \text{ and}$$

$$fe\frac{4\pi}{3}R0^3 FM = \frac{4\pi}{3}\left(\frac{hc}{2\pi}\right)^3\left(\frac{G_{geo}}{m_f mec^2}\right)^2 \equiv gF_{geo},$$

$$\text{where } fe = \frac{\pi^2}{4} \text{ and } G_{geo} = \frac{\pi}{4}\frac{Q}{3}g_e S\sqrt{\alpha fs}, \tag{11a}$$

or

$$\text{with } m_f \equiv \frac{Q}{3}m_{F0} \text{ to satisfy charge universality,}$$

$$\text{then if } m_{F0} \equiv \frac{m_{W-boson}}{me},$$

$$\text{we produce } GF_{geo} \equiv \frac{4\pi}{3}\left(\frac{hc}{2\pi}\right)^3\frac{1}{(m_{W-boson}c^2)^2} \times \left[\left(\frac{\pi}{2}g_e S\right)^2\frac{\alpha fs}{4}\bigg|_{S=1}\right] = GF;$$

$$\mu_{spin} \equiv \left(\frac{g_e}{2}\frac{Qe}{3M}\right)S\frac{h}{2\pi} \quad \text{and} \quad g_e = 2.00231930436. \tag{11b}$$

The quantity $fe = \pi^2/4$ represents an effective radial-, or volume-, factor specified to represent the "geometric" mass-energy distribution and, as quantified here, produces precise numerical agreement with *GF*. The relationship for the distortional-geometric-Fermi-constant is satisfied with a Fermi-mass (boson)-energy of 80.39664543913 GeV (calculated to the precision of the electron magnetic moment [14]). For experimental comparison, the boson-mass-energy is cited as "*Mass* = 80:399 ± 0:023 GeV" [14]. We have interpreted the geometric manifestation of the "Fermi-energy-volume product (GF)" as the condition of distortionally-produced "maximum curvature" in the geometric manifold; the "maximum curvature" producing element is the energy–density sum-quantity $Td_2^2 + Td_1^1$.

Table 1 Distortional-particle descriptors, charge-states and masses

Distortion	*R*0 (Fm)	κ (10^{-6} m/J)	*U/M*	Spin (μ_B)	*Q* (E)	*M* (me c^2)
Down-quark	2.44	22.1	1.006	0.0359	−1/3	9.29
Up-quark	6.66	262	1.009	0.123	2/3	5.405
Electron	47.2	1.72×10^4	1.012	1.00	−3/3	1.00
Muon	0.228	0.403	1.012	4.84×10^{-3}	−3/3	206.77
Tauon	0.0136	1.42×10^{-3}	0.9938	2.88×10^{-4}	−3/3	3,477
Neutrino	1.52×10^{-4}	3.202×10^4		5.4×10^{-11}	0	3.9×10^{-6}

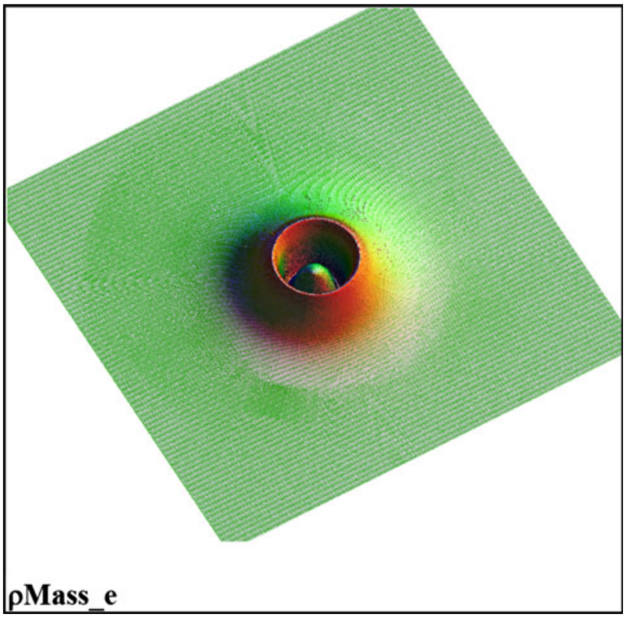

Fig. 3 Mass–energy–density distribution-function ($\rho Mass\_e$) surface-plot (view 1) for the particle-2 distortion (also see Fig. 2 and Eq. 3); linear radii and logarithmic amplitudes

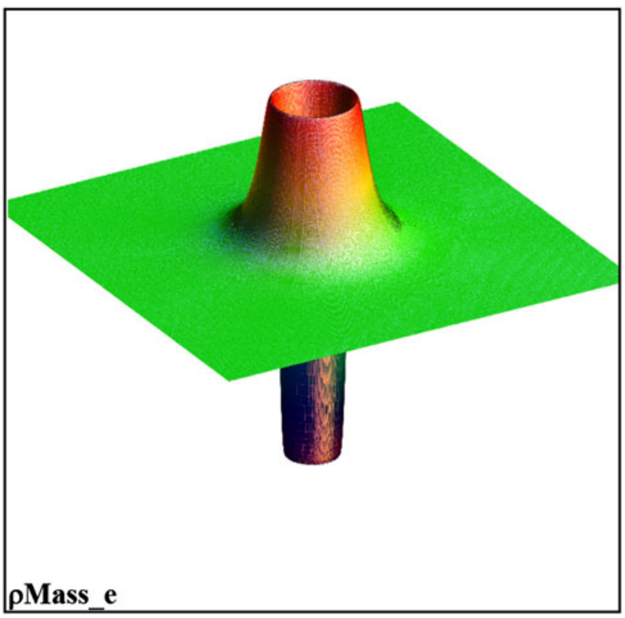

Fig. 4 Mass–energy–density distribution-function ($\rho Mass\_e$) surface-plot (view 2) for the particle-2 distortion (also see Fig. 2 and Eq. 3); linear radii and logarithmic amplitudes

4. Geometrostatic gravitational distortions

A more inclusive treatment of the total mass energy–density includes a gravitational contribution in addition to the electromagnetic component. That is, according to Eq. (12), we can write for the field-energy-densities at flat-space radii,

$$u_{geo-total} \equiv Fd^2_{14}(r \to \infty) \cong \frac{\kappa_{geo}}{4\pi r^4}(Mc^2)^2 = \frac{2\kappa o}{8\pi r^4}(Mc^2)^2 + \frac{G/c^4}{8\pi r^4}(Mc^2)^2. \quad (12)$$

The geometrical mimicking process produced an expanded composite coupling-constant $\kappa_{geo} = \kappa o + \kappa_G/2$ and $Rs_{geo} = 2Mc^2(\kappa o + \kappa_G/2)$. Consequently, both Maxwellian and gravitational physical-structures are geometrically mimicked with a single compound coupling-constant. Multi-dimensional space–time manifolds have been and are being studied in this regard [16–18].

It is posited that the radial zero, $u_0 \equiv u(r_0) = 1.63756$ if $r_0 \equiv Rs_{geo} = R0_{geo}/u_0$, in the distortional solution's field-tensor energy–density-distribution, is the geometric manifestation of the Schwarzschild "metric-radial-zero" (the radial singularity classically interpreted as a "black-hole" radius). The radial zero, from Eq. (5), is expressed as

$$\frac{1}{f}f' + \frac{1}{2f}(3-2f)\mu' + \frac{1}{r}(1-f) = 0 \quad \text{or } (1-3u^3)(1-u^3)^2 - 4u^2(Iu-\gamma) = 0. \quad (13)$$

The quantized geometric u_0 feature is interpreted as a quantized radial- or curvature-feature since electric-charge in this geometric model arises exclusively from the negative-energy–density core. However, the negative-energy–density region ($\gamma = 3.27512$ ($\gamma_0 \to 2.86398$)) of the distortional-field construct is present even in the absence of the "electric-charge-based" component and is interpreted as, and constitutes, a "gravitational" core-element. Since both Maxwellian and gravitational structures are manifested in the single geometric distortion, the gravitational structure is the ground-state interpretation, or the zeroth-order state, of the "geometric-distortional-charge"; the "Q *quantum number*" is thereby extended to the integer-range 0, 1, 2, 3.

5. Conclusions

It has been posited in the present work that the classical Riemannian four-dimensional curvature equations can be applied to describe localized geometrical distortions and associated energy distributions at quantum level magnitudes and distances. By requiring that geometric distortions mimic the physical characteristics of elementary particles, a global coupling constant between energy and geometry is produced. The theoretical modeling and calculational procedure is limited to those geometric-distortional families satisfying the equation-of-state, $Td^4_4 = -(Td^1_1 + Td^2_2 + Td^3_3)$, which also express static, spherically-symmetric Maxwellian tensor behavior. Functional solutions to the

differential equations describing the "distorted" space are of such a character that, over portions of the radial extensions of the distortion, the geometrical tensor elements exhibit negative, as well as positive, curvature-magnitudes and energy-densities. Within the "core regions" of the distortion, the propagation velocity exceeds the velocity of light (equivalent to an $n < 1$ refractive index). The field-observable in the negative energy–density spatial region (the core region) is non-Coulombic and non-infinite at the radial origin. This quantized negative-energy–density region is exclusively a function of geometrically-mimicked electric-charge and geometrically-mimicked gravitational-mass. Physical characteristics have been simulated for the fundamental particles as well as for hypothetical beta-decay transition-mediating distortion-particles.

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